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Just What is a Logarithm, Anyway?

By: Kim Aaron, Jet Propulsion Lab., 4800 Oak Grove Dr., Pasadena CA 91104
e-mail: kim.aaron@jpl.nasa.gov

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Before we talk about logarithms, let's think a little bit about graphing.

When we make a graph, usually the numbers on the axis go up in equal steps. Suppose we made a graph of the time it took for each student in the class to get home. We might label the ticks on the axis 0, 5, 10, 15, 20 and so on. The name of the axis might be "Time (minutes)." For each tick, we added the same amount each time. In this case, when we move one tick, we add 5 minutes. Suppose Cindy takes seven minutes to get home. We would put a mark on our graph part way between the 5 minute tick and the 10 minute tick. We don't have to line up our graph with the ticks exactly. We use the ticks on the axis like a ruler. We can measure in between the ticks too. In fact, sometimes we put little ticks in between the big ticks, but we don't put numbers next to them. Think about clocks. Sometimes each minute is marked, but only hours get printed numbers. The point here is that any time we move the same distance along the axis, we **ADD** the same amount to the thing we are graphing. This is true even when it is a distance that doesn't line up with the tick marks. This kind of axis or scale is often called a "Linear Scale" as shown in Figure 1.

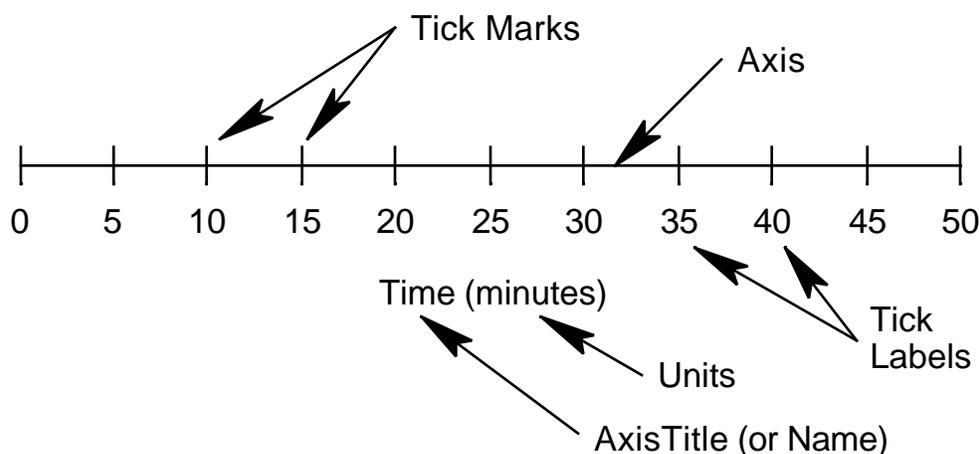


Figure 1 A Linear Scale

OK, all that is talking about normal graphing or measuring. Sometimes the thing we want to graph covers a huge range of sizes. Suppose we want to show on the same graph the time it takes Cindy to get home, the time it takes to drive across the entire country, the time for the earth to go around the sun, and the time it takes light to go from our sun to another galaxy. Oh, and maybe we also want to add the time it takes for a heartbeat too. Well, heartbeats take about a second. So maybe we could make a scale in seconds. We could put a point at 1 for the heartbeat. Then we'd need a

point at 420 for the time it takes Cindy to get home (7 minutes = 420 seconds). Then to drive across the country might take a week. that's about 600,000 seconds. Whew, it's already getting to be a pretty long graph! Next, we have the sun going around the earth in a year. That's a bit more than thirty million seconds. Now for sunlight to reach another galaxy could be millions of years. That's millions of millions of seconds. We just can't make a graph big enough to show all these on a regular (linear) scale. So what do we do?

When we make our graph, instead of ADDING as we move along the scale, we can MULTIPLY instead. I've created a graph (Figure 2) showing the various times I mentioned above. I've labeled the ticks in seconds: 0.1, 1, 10, 100, 1000, 10,000, and so on up to 1,000,000,000,000. Altogether, we have 14 labels along the axis. The last few numbers take a little room to write, but we can actually get them all onto one graph. In this example, I put the axis up and down instead of across so I could fit the numbers in more easily. A scale such as this is called a logarithmic scale. But we still haven't said what a logarithm actually is.

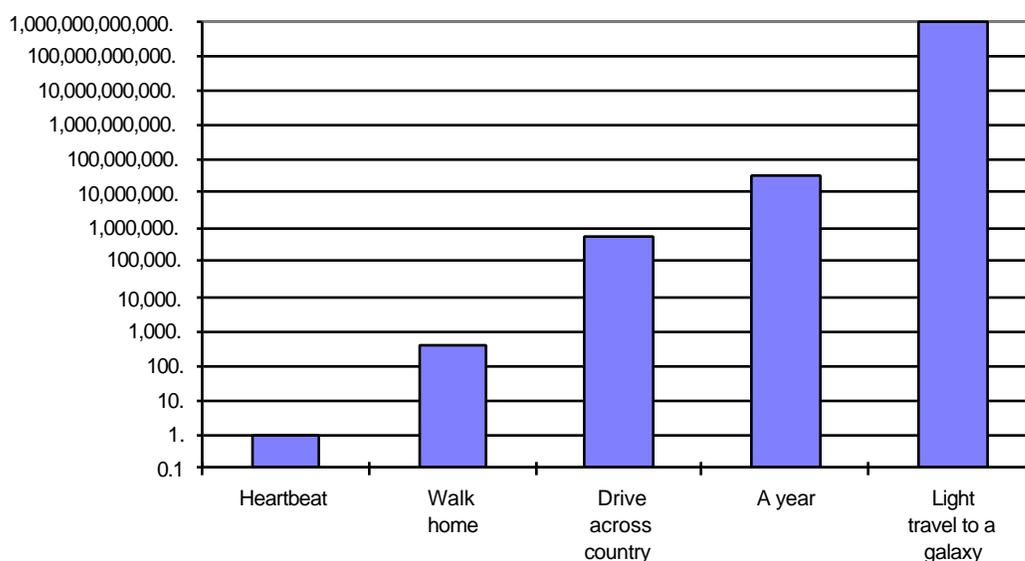


Figure 2 Example Graph using a Logarithmic Scale

You have probably seen exponents, such as $10^2 = 100$, or $10^6 = 1,000,000 =$ a million. We could write our tick labels more compactly using exponents: 10^4 , 10^5 , 10^6 ,... Those little superscripts (or numbers written above the line) are actually the logarithms of the big number. So the logarithm of a million is 6. The logarithm of 100 is 2. The logarithm of a number is how many times you have to multiply 10 by itself to get that number:

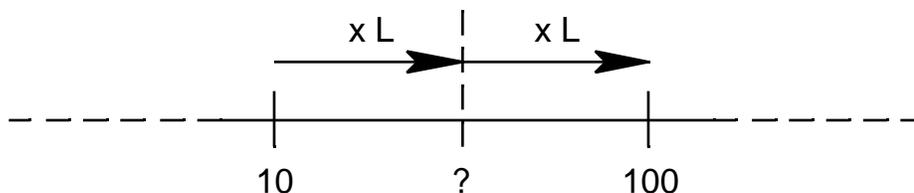
$$10,000 = 10 \times 10 \times 10 \times 10 = 10^4$$

$$\text{Logarithm of } 10,000 = 4$$

Notice that the logarithms of the numbers on our graph go up evenly. From one number to the next on our logarithmic scale, the logarithm goes up by one. That's why it's called a logarithmic scale. Well that's fine for numbers like 100 or 1000 where we multiply by 10 an integer number of times. But what about the numbers in between? What's the logarithm of 73 for instance?

Let's think about our two kinds of graphs: the normal one (which scientists often call a linear scale) and the logarithmic one. How do they behave for numbers in between the ticks? For the normal scale, we can put numbers in between without too much trouble: 2.5 would be half way

between 2 and 3. Each time we move half a tick length, we ADD 0.5 to the number. Now on the other scale, (the logarithmic scale) each time we move a tick length, we MULTIPLY by the same amount. Well, this is also true for numbers in between the ticks. Think about a point half way between 10 and a 100 on the graph. What number would it be? Take a guess. What number do you think it might be? We'll find out soon, but I bet you don't guess it right.



Going along the graph from 10 to a point halfway to 100, we multiply by some number. We'll call that number "L" for now. Then we'll figure out what L is. Next, going from this halfway point the same distance along the graph to 100, we multiply by the same number L again. So we multiplied 10 by L and then multiplied the result by L again to get 100. We could write it mathematically as $10 \times L \times L = 100$. This is Algebra, and not all of you have taken Algebra yet, but you could still think about trying different numbers for L until you found the right one to solve the equation. You could use a calculator to try different values for L until it works out close enough. You might call this a "trial-and-error" method. Actually, engineers and scientists often use this method to solve more complicated problems, although they might call it "iteration" instead.

But we can solve this equation directly. First divide both sides by 10 so we get $L \times L = 10$. You probably know that when you multiply a number by itself, it's called squaring the number. In this case, L squared is 10, or $L^2 = 10$. We also say L is the square root of 10. If you have a calculator, it probably has a square root button labeled $\sqrt{\quad}$. Try it. Enter 10 and then press the square root button. You should get 3.16227766. Some calculators won't show that many digits, but you get the idea. When you multiply $3.16227766 \times 3.16227766$ you should get 10. So we have solved our equation: $L = 3.162\dots$. This happens to be one of those irrational numbers, so the digits just keep on going.

Let's go back to our logarithmic scale. We often call it a "log scale" for short. That's much easier to say, and to type, too! On our log scale, we were trying to figure out what the number would be halfway between 10 and 100. We started by saying it is 10 times L and we just figured out that L is 3.162... so the number halfway between 10 and 100 on a log scale is 31.62. Did you guess it right? The same thing works half way between 100 and 1000. It is 316.2. That makes sense. We moved the same distance along the graph as the distance between ticks, so we just multiplied by 10.

Remember the logarithms are spaced equally on our log scale? The log of 10 is 1. The log of 100 is 2. The log of 1000 is 3. So what do you suppose the log of 316.227766 is? On a log scale, it's half way between 100 and 1000, so its logarithm is half way between 2 and 3. Its logarithm is 2.5. Check it on a calculator if it has a log button. Enter 316.227766 and press the log button. You should get 2.5 or something very close to 2.5.

But this also means that 10 to the power of 2.5 equals 316.227766. If you have a calculator with a button labeled x^y you can check it. It's a little hard to think about multiplying a number by itself two and a half times, but if you could do that, you'd get 316.227766. We use logarithms to let us do something equivalent to that. It's a bit strange, but it works, and it is very useful.

This is just the beginning of explaining what a logarithm is. I hope you begin to get the idea that it is **useful for working with numbers whose sizes change a lot.**

Examples from my work

As an engineer, I have used logarithms in my work many many many times. It was a hard idea to understand at first, but it's something I really needed to understand and something I use all the time.

For one of my jobs (actually my first job as an engineer), I had to design a special nozzle to create **rain drops** to use in a wind tunnel to see how much rain affects the wings of airplanes. Now raindrops are not all the same size. Some are actually very small and only a few are the big blobs that splatter so much in a heavy downpour. Well, the sizes vary so much that I had to make graphs of their sizes on a log scale.

When I was a student at college, I did some acoustics experiments. Acoustics deals with **sound**. The sound pressure that our ears can hear is incredibly small. But the louder sounds we listen to are many times bigger. The whole range is something like the range of our graph of going from 1 to a million million that we talked about above. So when we measure sound, we don't usually talk about the actual pressure. Instead, we talk about the logarithm of the pressure. For sounds, this logarithm has a special name: Sound Pressure Level (SPL) and it is measured in decibels. That is just another way of using logarithms. Maybe you will learn more about decibels later in school, especially if you go to an engineering or science college.

Right now I am designing a **vibration isolation system** for an experiment that we hope will fly aboard the International Space Station in a few years. The accelerations that it will experience vary a lot. When this thing is sitting on the ground, it feels an acceleration of one "gee," or one "gravity," the same as you feel. When it gets launched on the Space Shuttle, it is on something that shakes around a whole lot. Then it feels as much as a hundred gees (it weighs a hundred times more than normal). Don't worry, the astronauts don't feel that much, it's just that the experiment is fastened to something that shakes around a lot. Once the experiment gets into orbit around the earth, the accelerations are as small as a few millionths of a gee, or a million times smaller than the acceleration on the earth (it is very nearly "weightless"). So in describing the accelerations that this gadget I'm building will feel, I need to use logarithms to cover the whole range. I couldn't possibly plot them all on a linear graph. Below is another example from the same work. It shows the **vibration levels in the Space Shuttle**. The labels on the vertical axis are in the format the computer uses for scientific exponential notation. The "E" means "times 10 to the Exponent." So $1.0E+04$ is the same as 1.0×10^4 . Don't worry too much about the units. By "ug" I mean "microgravity" or a millionth of earth's gravity. The vibration levels on the Space Shuttle are very low. Still, they cover a large RANGE of values, so I had to use a log scale.

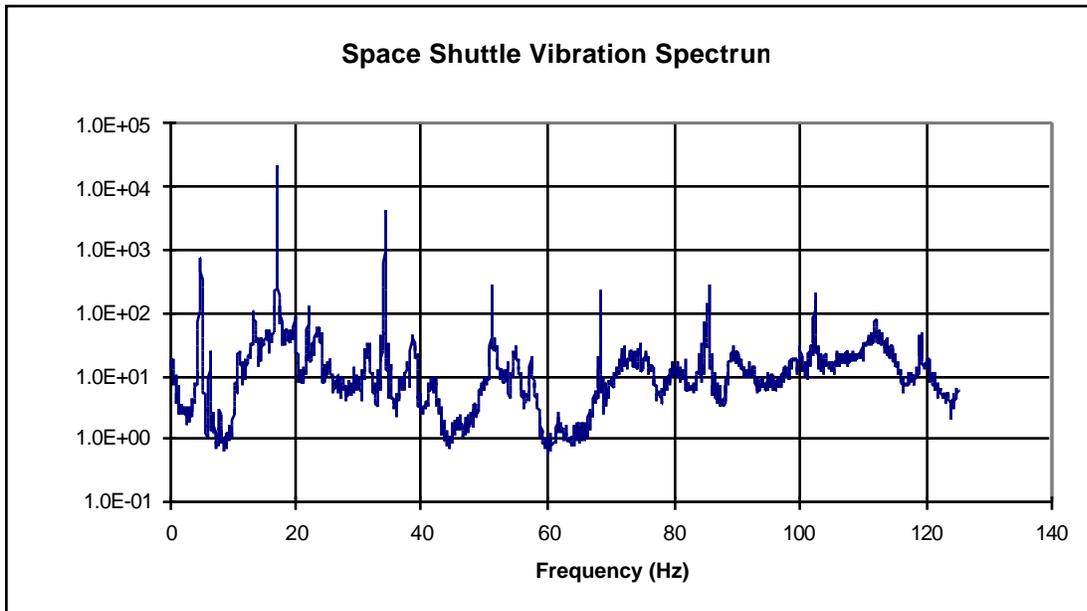


Figure 3 An example of a graph I used in my work. Note the log scale.

A more common example of a log scale is the **Richter Scale for earthquakes**. Each step on the Richter scale means the earth quake released ten times more energy. An earthquake of magnitude 6 is ten times worse than an earthquake with magnitude 5. A geologist would give a more precise definition, but the idea is the same. Each step on the log scale corresponds to multiplying by 10.