

Real-time atmospheric turbulence profile estimation using modal covariance measurements from multiple guide stars

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ABSTRACT

An accurate and timely model of the atmospheric turbulence profile is an important input into the construction of tomographic reconstructors for laser tomography adaptive optics (LTAO) and multi-conjugate adaptive optics (MCAO) using multiple guide stars. We report on a technique for estimating the turbulence profile using the correlations between the modal reconstructions of open-loop wavefront sensor (WFS) measurements from natural or laser guide stars. Laser guide stars can provide an estimate of the turbulence profile along the line of sight to any suitable science target. Open-loop WFS measurements, acquired at the MMT telescope, have been analyzed to recover an estimate of the C_n^2 profile. This open-loop WFS data can be used to yield turbulence estimates in near real-time, which can be used to update the tomographic reconstructor prior to closed-loop operation.

This method can also be applied in closed-loop, using telemetry data already captured by multi-guide star adaptive optics (AO) systems, by computing estimates of the wavefront modal covariances from the closed-loop WFS residual error signals and the deformable mirror (DM) actuator positions. This will be of particular value when implemented with accurate position feedback from the AO system's DMs, rather than the input actuator commands, as is possible with an adaptive secondary mirror. We plan the first tests of the technique with the MMT's adaptive secondary and five Rayleigh laser guide stars.

Keywords: adaptive optics, atmospheric turbulence, tomography

1. INTRODUCTION

An accurate model of the atmospheric turbulence profile is an important input to the process of building tomographic reconstructors for laser tomography adaptive optics (LTAO) and multi-conjugate adaptive optics (MCAO) using multiple guide stars. Turbulence models are often produced using seasonally averaged measurements from equipment located at a nearby station. Obtaining a near real-time estimate for the turbulence profile, in the direction of a specific science target, will be critical to optimize the performance of these reconstructors in closed-loop operation. A detailed knowledge of the distribution of turbulence along a particular line of site will also permit the selection of the optimal science program and observing mode for the given seeing conditions.¹ In addition, since the measurements are made using the same instrumentation that is making the science observations, local effects such as dome seeing are correctly incorporated into the turbulence profile estimate.

Various techniques, such as SCIDAR, MASS, and DIMM,² have historically been used to characterize the seeing conditions at an astronomical site. These methods all require specialized equipment and in some cases are unable to resolve the turbulence profile near the ground. Recently, refinements to these methods such as G-SCIDAR^{3,4} have been successfully used at sites like Mt. Graham, Arizona to obtain very detailed measurements of the C_n^2 profile including measurements of the important boundary layer.

The SLODAR technique⁵ can be used to estimate the C_n^2 profile using the spatial cross-correlations of Shack-Hartmann wavefront sensor slope measurements from a binary star. This technique has the advantage of not requiring additional equipment, beyond a natural guide star (NGS) wavefront sensor (WFS). However, the turbulence profile can only be estimated in the direction of a bright and suitably separated binary star source and the number of heights estimated is determined by the number of subapertures across the diameter of the WFS pupil. The maximum height that can be measured is determined by the separation of the binary source.

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In this paper, we report on a related approach to estimating the C_n^2 profile using the correlations between the modal reconstructions of WFS measurements from multiple natural or laser guide stars. This method takes advantage of the WFS telemetry data already captured by multiple guide star adaptive optics (AO) systems and estimates the turbulence profile, in near real-time, over a range of heights determined by the number of guide stars and the number of reconstructed modes. If laser guide stars (LGS) are used, this measurement can be performed anywhere on the sky and can provide an estimate of the turbulence profile along the line of site to a specific science target.

A closed-loop multiple laser guide star adaptive optics system is currently being commissioned at the 6.5 m MMT telescope at Mt. Hopkins.⁶⁻⁸ The current status of the MMT laser AO system is described elsewhere in these proceedings.⁹

The MMT laser system projects five Rayleigh LGS from behind the adaptive secondary mirror of the MMT, producing a regular pentagon of spots on the sky with a radius of 60 arc-seconds, a nominal height of 23 km, and with a total output of 25 W of 532 nm light. The WFS instrumentation system generates a real-time telemetry stream of slope measurements from each of the five laser beacons. In open-loop, these measurements represent the integrated atmospheric effects along five lines of sight and, despite the cone effect, will provide correlated samples of the atmospheric turbulence up to a height of 10 km. With a smaller, 20 arc-second diameter, constellation of laser guide stars, more suitable for LTAO and MCAO observations, this technique would be able to estimate a turbulence profile to a maximum height of 17 km at the MMT.

Measurements have been made of open-loop wavefront slopes at frame rates of 208 Hz (April 2006) and 460 Hz (April 2007) using the constellation of five Rayleigh LGS at the MMT telescope. These slope measurements have been used to estimate the turbulence profile at the time of those observations.

2. MODAL COVARIANCES FOR MULTIPLE BEACONS

2.1 Analytic Covariance Calculation

Analytic expressions for the modal covariance between multiple beacons have been developed for natural guide stars¹⁰ as well as for laser beacons.¹¹ The most general is Whiteley's expression, shown in equation 1, which allows for multiple telescope apertures and fully accounts for the finite height of laser beacon sources.

$$\begin{aligned}
B_{a_1 a_2 j l}(\mathbf{r}_{a_1}, \mathbf{r}_{s_1}; \mathbf{r}_{a_2}, \mathbf{r}_{s_2}; z_l) = & \\
& [\pi R_1 R_2 (1 - A_{1l}) (1 - A_{2l})]^{-1} [(n_i + 1)(n_j + 1)]^{\frac{1}{2}} (-1)^{\frac{1}{2}(n_i + n_j)} 2^{1 - \frac{1}{2}(\delta_{m_i 0} + \delta_{m_j 0})} (-1)^{m_j} \times \\
& \left\{ \left\{ (-1)^{\frac{3}{2}(m_i + m_j)} \cos \left[(m_i + m_j) \theta_{s_l} + \frac{\pi}{4} \left[(1 - \delta_{m_i 0}) \left((-1)^i - 1 \right) + (1 - \delta_{m_j 0}) \left((-1)^j - 1 \right) \right] \right\} \times \right. \\
& \left. \int_0^\infty \frac{dx}{x} W_{\phi_l} \left(\frac{x}{2\pi} \right) J_{m_i + m_j} [s_l x] J_{n_i + 1} [R_1 (1 - A_{1l}) x] J_{n_j + 1} [R_2 (1 - A_{2l}) x] \right\} + \quad (1) \\
& \left\{ (-1)^{\frac{3}{2}|m_i - m_j|} \cos \left[(m_i - m_j) \theta_{s_l} + \frac{\pi}{4} \left[(1 - \delta_{m_i 0}) \left((-1)^i - 1 \right) - (1 - \delta_{m_j 0}) \left((-1)^j - 1 \right) \right] \right\} \times \right. \\
& \left. \int_0^\infty \frac{dx}{x} W_{\phi_l} \left(\frac{x}{2\pi} \right) J_{|m_i - m_j|} [s_l x] J_{n_i + 1} [R_1 (1 - A_{1l}) x] J_{n_j + 1} [R_2 (1 - A_{2l}) x] \right\} \left. \right\}.
\end{aligned}$$

This expression assumes an atmospheric model with discrete turbulent layers indexed by l at heights z_l and with turbulence strength $C_{n,l}^2$. $B_{a_1 a_2 j l}$ is the covariance between Zernike mode i , measured within aperture a_1 , and Zernike mode j , measured within aperture a_2 , caused by a turbulence at layer l . Zernike modes i and j have radial and azimuthal orders (n_i, m_i) and (n_j, m_j) respectively. \mathbf{r}_{a_1} and \mathbf{r}_{a_2} are vectors from a common coordinate system origin to the centers of apertures a_1 and a_2 . These apertures have pupil radii R_1 and R_2 respectively. \mathbf{r}_{s_1} and \mathbf{r}_{s_2} are vectors to the sources s_1 and s_2 . A_{1l} and A_{2l} are the ratios of the radii of the source light cones at the intersection with turbulent layer l to the corresponding pupil radii at the aperture. s_l is the distance between the center lines of these two light cones at the layer height z_l .

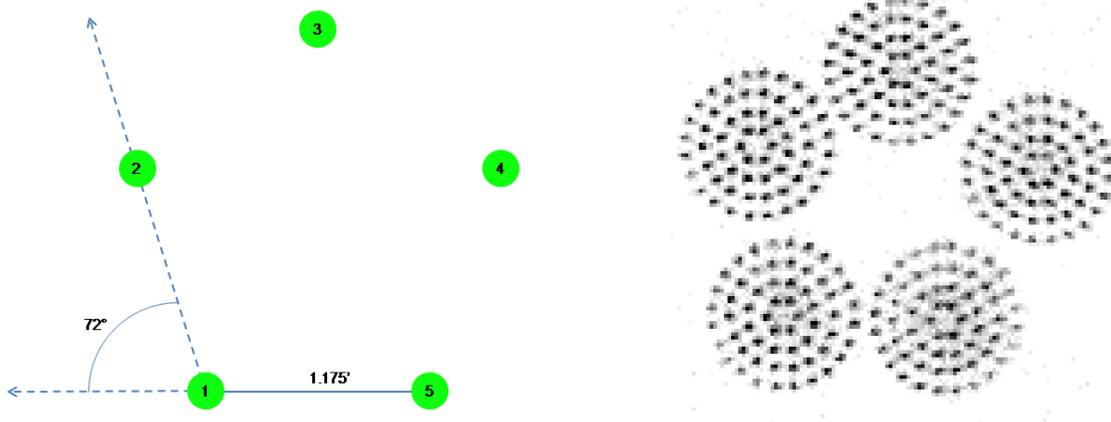


Figure 1. (Left) Separation and relative orientation angles of adjacent MMT LGS beacon source pairs. (Right) Image of WFS Hartmann spots for five LGS beacons captured on a single CCD frame using the hexapolar geometry of the MMT LGS WFS prism array.

Finally, the spatial frequency is $k = x/2\pi$ for the von Kármán phase power spectral density at wavelength λ which, ignoring the inner scale, is

$$W_{\phi_l}(k) = 0.033C_{n,l}^2 \left(\frac{2\pi}{\lambda}\right)^2 (k^2 + k_0^2)^{-11/6} \quad (2)$$

where $k_0 = 1/L_0$ for outer scale L_0 (without a factor of 2π using Whiteley's conventions).

For a simpler geometry with a pair of laser beacons with a common source height and for a single telescope aperture we can simplify equation 1 and write, for the modal cross-correlation $B_j(z_l)$ of Zernike mode j ,

$$B_j(z_l) = (\pi R_l^2)^{-1} (-1)^{n+m} 2^{1-\delta_{m0}} (n+1) \int_0^\infty x^{-1} W_{\phi_l}(x/2\pi) J_{n+1}^2(R_l x) [J_0(s_l x) + K_j J_{2m}(s_l x)] \quad (3)$$

where

$$K_j = (-1)^{m+j(1-\delta_{m0})} \cos(2m\theta_{s_l}), \quad (4)$$

(n, m) are the radial and azimuthal orders of Zernike mode j , and the pair of beacon cones intersecting the layer l each have radii R_l and vector separation $\mathbf{s}_l = (s_l, \theta_{s_l})$. The functions J_μ are the Bessel functions of the first kind of order μ and δ_{nm} is the Kronecker delta which is 1 if $n = m$ and zero otherwise.

We can obtain an estimate of L_0 from open-loop WFS measurements, either from an NGS WFS or the ground layer estimate from multiple LGS WFS, of the rms amplitudes of Zernike modes from orders 1 to 8 by solving the non-linear system of equations in r_0 and L_0 from Chassat.¹² Then we can form a linear system of equations in $C_{n,l}^2$ and solve for fractional C_n^2 at a set of discrete layers l . Finally, using our estimate of r_0 , we can convert the fractional C_n^2 estimates to a full C_n^2 profile.

2.2 MMT Beacon Geometry

We use equation 3 to estimate the heights containing significant turbulence strength from open-loop data acquired at the MMT multiple LGS system. The MMT laser system projects five Rayleigh LGS from behind the adaptive secondary mirror of the MMT, producing a regular pentagon of spots on the sky with a radius of 60 arc-seconds. This produces a set of five pairs of sources around the outer circumference of the pentagon, shown in figure 1, with separation 1.175 arc-min and source pair orientation angles that are multiples of 72 deg.

The LGS WFS samples the pupil using 60 subapertures in a hexapolar geometry, also shown in figure 1, and generates an estimate of the amplitudes of the first 42 Zernike modes corresponding to radial orders 2 – 8. We

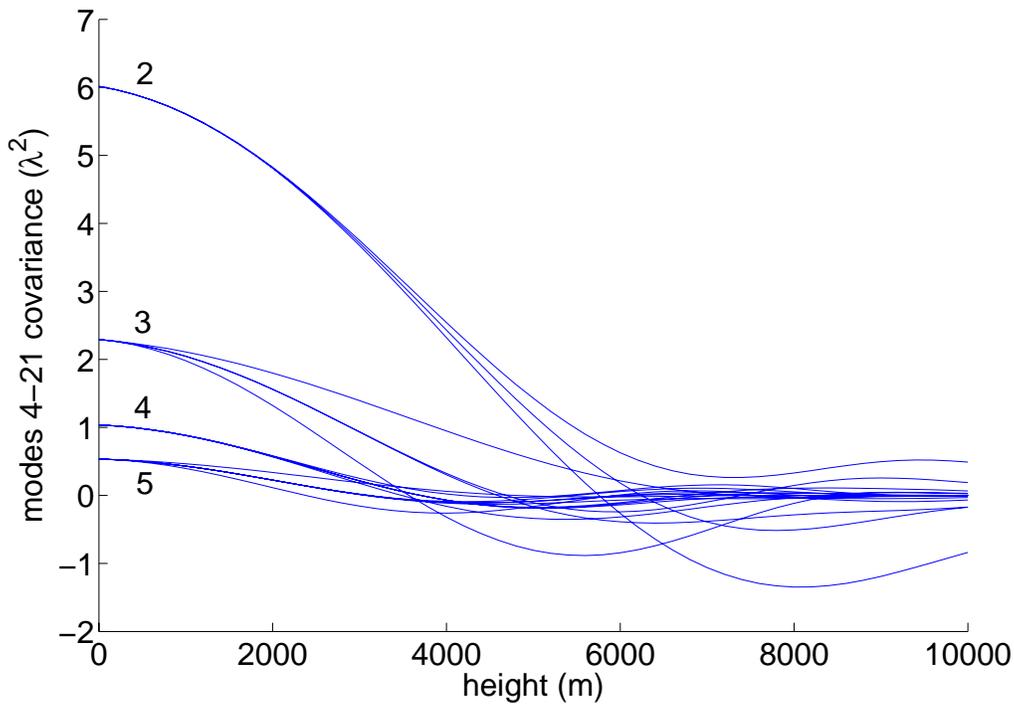


Figure 2. Analytic covariance curves as a function of layer height for Zernike modes 4 through 21 with the Zernike order number indicated for each group of curves.

use the lower radial orders 2 – 6 to avoid the increased noise in the estimates of the higher spatial frequency Zernike modes. With a nominal source height of 23 km, these sources have a maximum correlation height of 10.2 km and so we estimate the turbulence strength at 21 discrete heights at intervals of 500 m from 0 km to 10 km.

Figure 2 shows the model covariance profiles, as a function of layer height, for the Zernike modes 4 – 21 using equation 3. These model curves were generated for the first pair of LGS sources, sources 1 and 2 in figure 1, using $r_0 = 15.9$ cm and $L_0 = 13.5$ m which match the data obtained in April 2006.

The process of solving for the turbulence profile involves finding the linear combination of model curves, such as those shown in figure 2, that best match the measured modal covariances from WFS measurements.

2.3 Verification through Simulation

A Monte-Carlo simulation was performed to ensure that equation 3 accurately reflect the covariance between a separated pair of sources for a given Zernike mode. A series of ten thousand phase screens were generated using the von Kármán turbulence spectrum at a range of layer heights from zero to 10 km. The phase screens were sampled within the footprint of the intersection of the source light cones and each turbulent layer. Covariances, for a range of Zernike modes, were computed for this simulation and compared to the analytic model represented by equation 3.

Figure 3 shows the agreement between the analytic model and simulation for Zernike modes four and nineteen. Increasing the number of phase screens generated in the Monte-Carlo increases the correspondence between model and simulation. This simulation used the same source geometry and turbulence parameters as in figure 2.

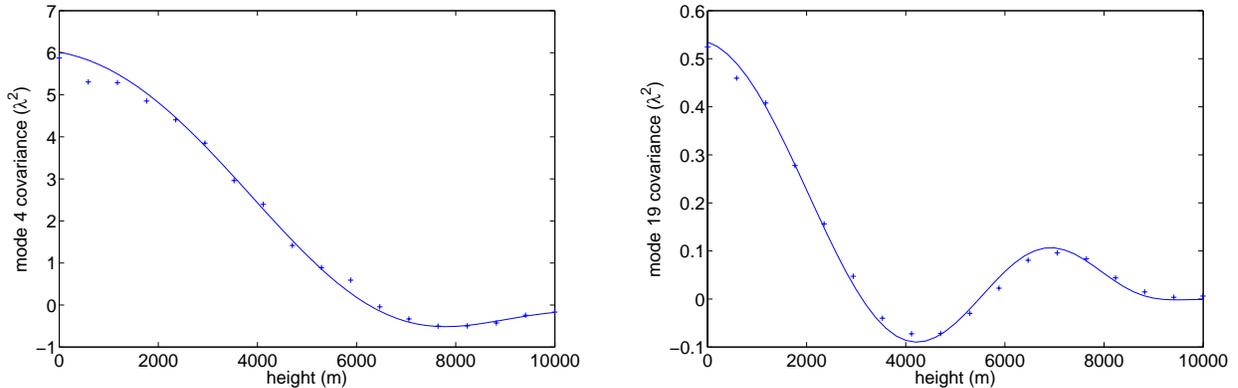


Figure 3. (Left) Analytic covariance curve (solid line) as a function of layer height for Zernike mode 4 vs. simulated covariance (crosses). (Right) Similar plot for Zernike mode 19.

3. ESTIMATING THE C_N^2 PROFILE FROM WAVEFRONT SENSOR DATA

3.1 Solving the Linear System

Consider a constellation of N guide stars from each of which Z Zernike modes are reconstructed. One can then compute $K = ZN(N - 1)/2$ values of the correlations $B_j(z_l)$ in equation 3 which may be written as a column vector M_k with $k = 1..K$. Since equation 3 depends linearly on $C_{n,l}^2$, it may be recast in the form of a simple $K \times L$ linear system

$$\sum_{l=1}^L B_{kl} C_{n,l}^2 = M_k \quad (5)$$

where B_{kl} is the correlation coefficient from equation 3 after factoring out the turbulence strength. The values of B_{kl} may be computed analytically with knowledge of the beacon geometry, and equation 5 may then be inverted and solved for the values of $C_{n,l}^2$ for each of the L layers.

We can, in principle, solve for $C_{n,l}^2$ at $L = ZN$ layers, corresponding to the number of independent measurements made on the WFS.

3.2 Solution using Optimization

However, this linear system is poorly conditioned. Examining the model covariance curves in figure 2, we can see that there is only a subtle differentiation in the curves between modes of the same radial order at lower heights. Also, in order to differentiate the covariance at different heights, we depend on covariance measurements from higher spatial frequency modes which have a smaller amplitude and hence are more susceptible to noise in real WFS measurements.

In practice, we have had the best success so far, treating the system of equations 5 as an optimization problem and using a Nelder-Mead simplex method (implemented by MATLABTM) in combination with simulated annealing. We construct a merit function which minimizes the mean square difference between the measured covariance data and the model covariances subject to a positivity constraint. It is an area of ongoing research to find the optimization method that is best able to deal with the large number of local minima and complicated error surface of this problem.

Our major goal is to identify the layer heights containing significant atmospheric turbulence in order to build an appropriate tomographic reconstruction matrix¹³ for the current seeing conditions, along the line of sight (LOS) to the science target.

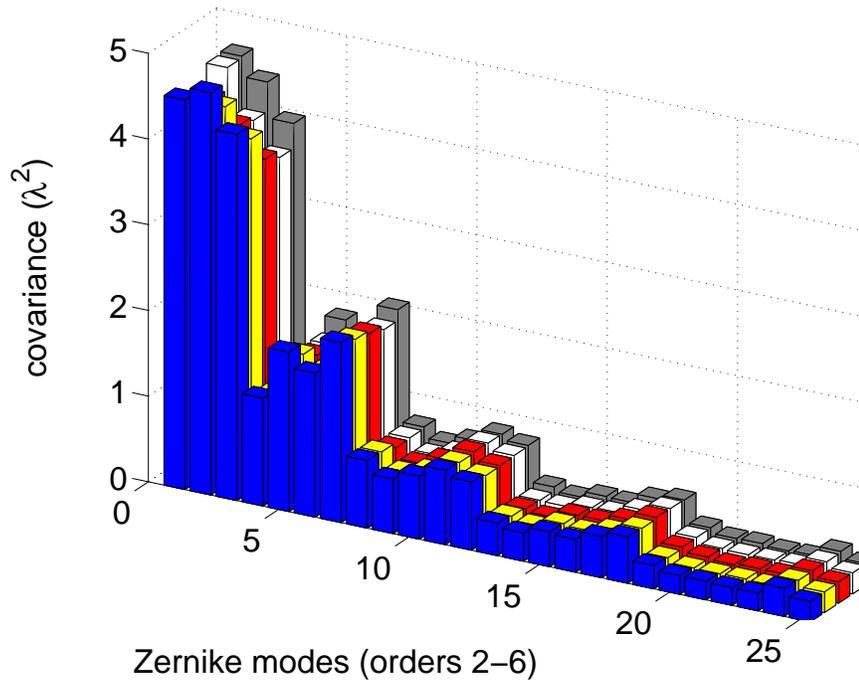


Figure 4. Measured covariance for the first twenty five Zernike modes starting with order two from 208 Hz measurements using the MMT multiple Rayleigh laser guide stars. Each row corresponds to a pair of LGS beacon sources with separation 1.175 arc-min. Modes decorrelate at different rates for different source pairs based on the separation and relative orientation of the source light cones.

3.3 MMT C_n^2 Profiles from Five Rayleigh Laser Beacons

Below, we analyze one minute duration datasets containing WFS measurements that were made from the MMT multiple LGS system in open-loop in April 2006 at 208 Hz and in April 2007 at 460 Hz. The following table lists the key parameters for each run.

Run	r_0 (cm)	L_0 (m)	Total C_n^2	Ele. (deg)
April 2006	15.9	13.5	3.2×10^{-13}	71.7
April 2007	16.2	20.1	3.1×10^{-13}	66.3

Figure 4 shows modal covariance measurements for each of the five adjacent LGS source pairs around the outer circumference of the pentagon of spots shown in figure 1. The differences in the relative amplitudes of the covariance measurements between the source pairs are the significant features used in the optimization process to find the best matching C_n^2 profile for these April 2006 measurements.

Figures 5 and 6 show the fractional C_n^2 estimates at 21 heights in 500 m increments at distance between zero and 10 km along the line of sight to the science target the during the 2006 and 2007 telescope runs. The results for the April 2007 run, shown in figure 6, have a similar profile to results obtained by McKenna, et. al.¹⁴ at the ridge adjacent to the MMT telescope in June 2002.

In both cases, over 60% of the turbulence strength is within the first 500 m of the ground. This is consistent with estimates obtained from the performance of the ground layer (GLAO) correction analysis for those runs.⁶⁻⁸

For the April 2006 observation, significant turbulence was found at 0, 1.5, and 9.5 km. During the April 2007 observation, significant turbulence was found at 0, 5.5, and 9.5 km. These measurements of discrete turbulence

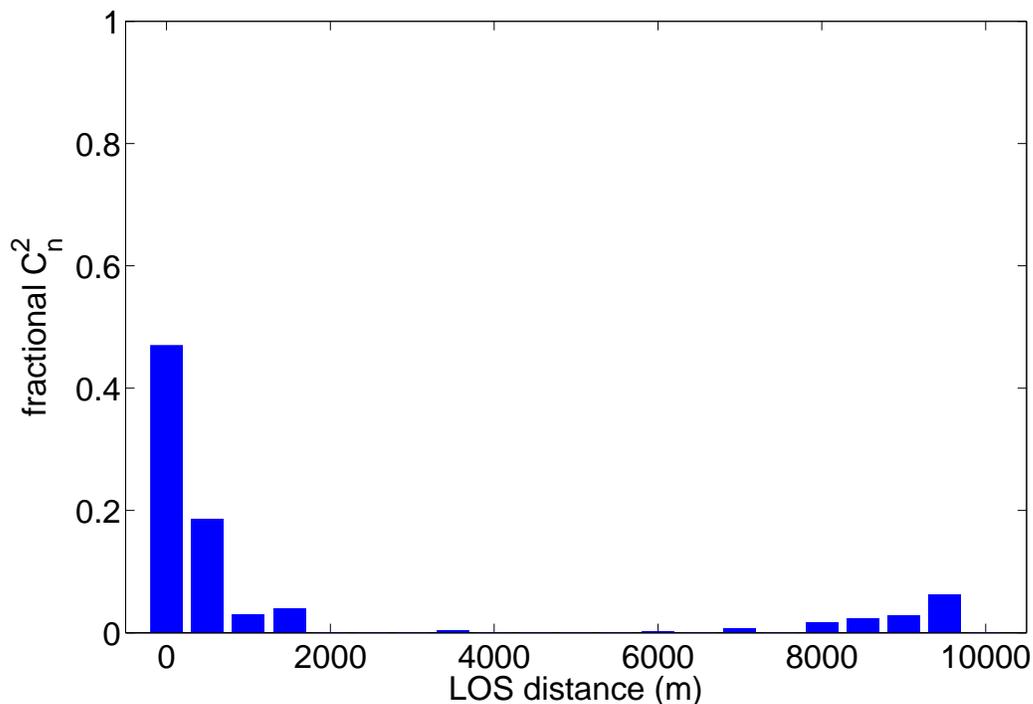


Figure 5. Fractional C_n^2 profile from 208 Hz measurements in April 2006 as a function of the distance along the line of sight (LOS) to the science target.

heights would be used to produce a three layer LTAO tomographic reconstructor, in each case, customized to current seeing conditions and with layer heights adapted to the line of sight and zenith angle to the science target. In simulation, reconstructor performance has been found to be only a weak function of small changes in layer height.¹³

4. NEAR REAL-TIME ESTIMATION OF TOMOGRAPHIC PARAMETERS

To date, this analysis has only been performed using open-loop WFS measurements made at the MMT telescope using the multiple LGS AO system. It is desirable to use closed-loop measurements in order to optimize science throughput at the telescope. Closed-loop measurements would also allow near real-time changes to the LTAO tomographic reconstructor matrix taking into account changes in the turbulence profile that occur during a long observation.

The LGS AO system at the MMT telescope has already implemented a real-time telemetry stream from both the WFS and the DM at the full frame rate of the AO control loop. WFS slope measurements, representing the residual error signals between the measured atmospheric turbulence and the current DM correction, are transmitted over the telemetry network and logged to disk during closed-loop operation. In addition, absolute position measurements for each of the 336 actuators on the MMT adaptive secondary mirror are part of the real-time telemetry stream. The MMT adaptive secondary, with its capacitive sensors at each actuator location,¹⁵ is unique in its ability to record absolute positions for the mirror surface during closed loop operation.

By combining the WFS residual error signals and the DM absolute position measurements, it is possible to accurately estimate the absolute amplitude of the atmospheric turbulence and produce the modal covariance measurements necessary to compute an estimated C_n^2 profile.

An updated LTAO reconstruction matrix can be calculated and loaded into the reconstructor control computer during closed-loop operation to optimize the AO performance to account for up to the minute seeing information

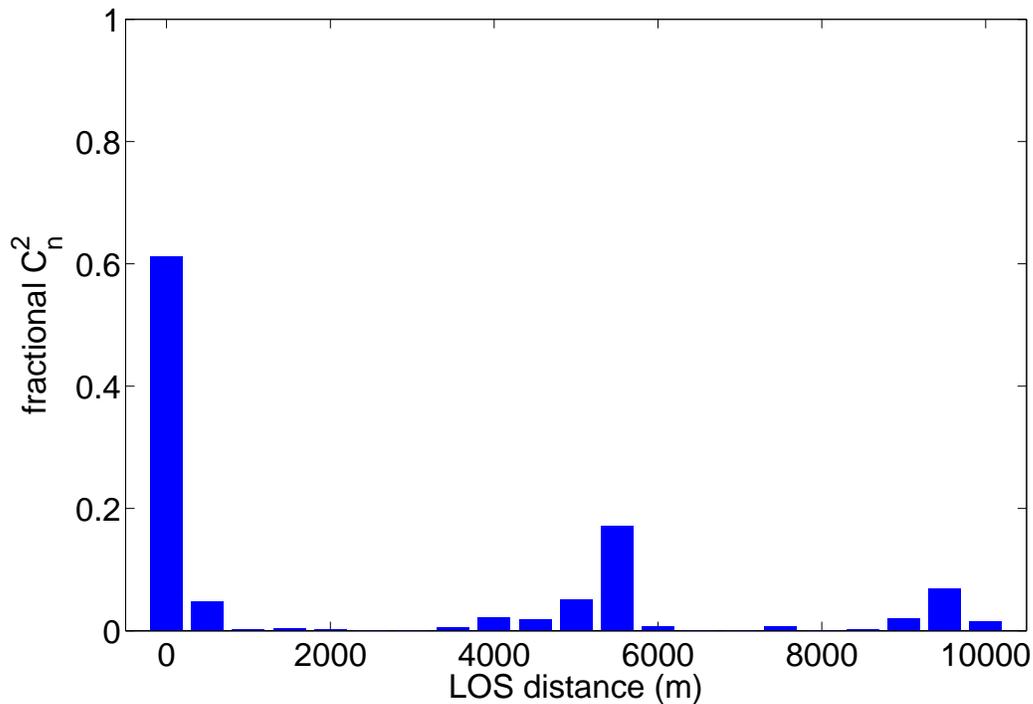


Figure 6. Fractional C_n^2 profile from 460 Hz measurements in April 2007 as a function of the distance along the line of sight (LOS) to the science target.

along the line of sight to the science target.

5. CONCLUSION AND FUTURE WORK

We have described and demonstrated a technique to obtain accurate and timely atmospheric turbulence profiles for use in the construction of tomographic reconstructors for LTAO and MCAO using multiple guide stars. These measurements can be made without any additional equipment, account for all local seeing effects, and provide layer distance estimates along the line of site to the science target. We have also discussed how this method can be applied directly in closed-loop, using telemetry data already captured by multi-guide star AO systems which have DMs that provide absolute position telemetry data.

The next important step for this analysis is to estimate C_n^2 profiles for WFS measurements in conjunction with simultaneous G-SCIDAR measurements to verify the accuracy of the reconstructed turbulence profiles.

We will also use this technique to obtain near real-time estimates of the C_n^2 profile during future closed-loop operations of the multiple LGS AO system at the MMT telescope. We will use these profiles to evaluate the effectiveness of our GLAO wide field compensation and to optimize the tomographic reconstructors used for the narrow field LTAO mode at the MMT.

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